MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 27-08-2021 (MORNING SHIFT) | TIME : (9.00 am to 12.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. In Millikan's oil drop experiment, what is viscous force acting on a uncharged drop of radius 2×10^{-5} m and density 1.2×10^{3} kgm^{-3} ? Take viscosity of liquid = 1.8×10^{-5} Nsm^{-2}. (Neglect buoyancy due to air).

(1) 5.8×10^{-10} N (2) 1.8×10^{-10} N (3) 3.8×10^{-11} N (4) 3.9×10^{-10} N

Ans. (4)

Sol. viscous force = weight

$$= mg = \rho x\left(\frac{4}{3}\pi r^3\right)\times g = 3.9\times10^{-10}\text{N}$$

2. A huge circular arc of length 4.4 ly subtends an angle 4s at the centre of circle. How long it would take for a body to complete 4 revolution if its speed is 8 AU per second ?

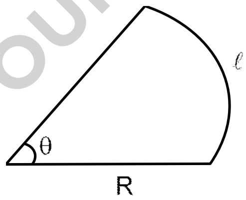
Given : 1 ly = 9.46×10^{15} m

(1) 4.1×10^{8}s (2) 4.5×10^{10}s (3) 7.2×10^{8}s (4) 3.5×10^{6}s

Ans. (2)

Sol. $\ell = 4.4\text{ly} = 4.4 \times 9.46 \times 10^{15}$

Length of Arc. = $\ell = R\theta$



$4.4 \times 9.46 \times 10^{15} = R\theta$

$\theta = 4S = 4 \times 4.83 \times 10^{-6} = 1.94 \times 10^{-5}$ rad

$4.4 \times 9.46 \times 10^{15} = R \times 1.94 \times 10^{-5}$

$R = 2.4155 \times 10^{21}$ meter

Speed = 8 Au = $8 \times 15 \times 10^{11}$ m/s = 12×10^{11}m/s

4 revolution means distance = $4 \times 2\pi R$ metre

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{4\times2\pi R}{12\times10^{11}};\text{time} = \frac{8\times3.14\times2.1455\times10^{21}}{12\times10^{11}} \Rightarrow 4.5\times10^{10}\text{ sec}$$

3. Which of the following is not a dimensionless quantity?

(1) Permeability of free space (μ_0)

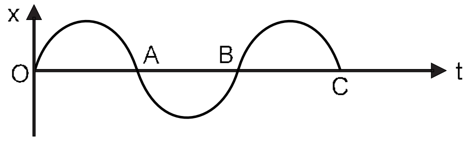
(2) Relative magnetic permeability (μ_r)

(3) Power factor

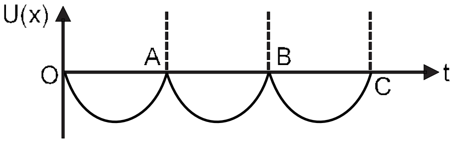
(4) Quality factor

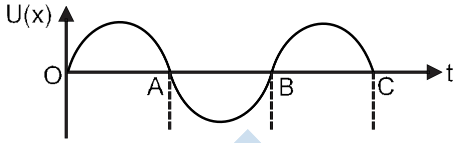
Ans. (1)

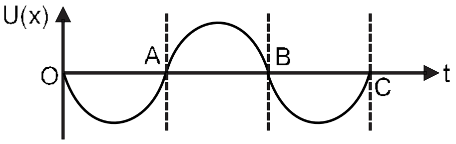
4. The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure.

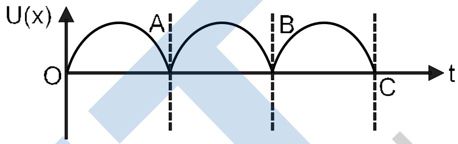


The potential energy U(x) versus time (t) plot of the particle is correctly shown in figure :

(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. Considering $\rightarrow$ Spring –mass system

$x = x_0 \sin \omega t$

$$\text{P.E.} = \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2 \sin^2 \omega t = c\sin^2 \omega t$$

$$\Rightarrow \text{PE} = \left(\frac{1-\cos 2\omega t}{2}\right)$$

5. A uniformly charged disc of radius R having surface charge density σ is placed in the xy plane with its center at the origin. Find the electric field intensity along the z–axis at a distance Z from origin :

(1) $E = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{Z^2+R^2}\right) + \frac{1}{Z^2}$

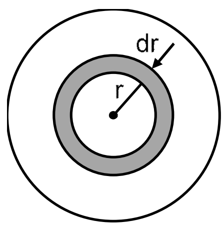
(2) $E = \frac{2\varepsilon_0}{2\sigma}\left(\frac{1}{(Z^2+R^2)^{1/2}} + Z\right)$

(3) $E = \frac{\sigma}{2\varepsilon\sigma}\left(1 + \frac{Z}{(Z^2+R^2)^{1/2}}\right)$

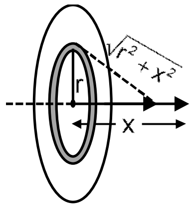
(4) $E = \frac{\sigma}{2\varepsilon\sigma}\left(1 - \frac{Z}{(Z^2+R^2)^{1/2}}\right)$

Ans. (4)

Sol. The disc can be considered to be a collection of large number of concentric rings. Consider an element of the shape of rings of radius r and of width dr. Electric field due to this ring at P is



$$dE = \frac{K.\sigma 2\pi r.dr.x}{\left(r^2 + x^2\right)^{3/2}}$$



Put, $r^2 + x^2 = y^2$

$2rdr + 2ydy$

$$\therefore dE = \frac{K.\sigma 2\pi y.dy.x}{y^3} = 2K\sigma\pi.x\frac{ydy}{y^3}$$

Electric field at P due to all rings is along the axis:

$$\therefore E = \int dE \Rightarrow E = 2K\sigma\pi x\int_x^{\sqrt{R^2+x^2}}\frac{1}{y^2}dy = 2K\rho\pi x.\left[-\frac{1}{y}\right]_x^{\sqrt{R^2+x^2}}$$

$$= 2K\sigma\pi x\left[+\frac{1}{x} - \frac{1}{\sqrt{R^2+x^2}}\right] = 2K\sigma\pi\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$$ along the axis

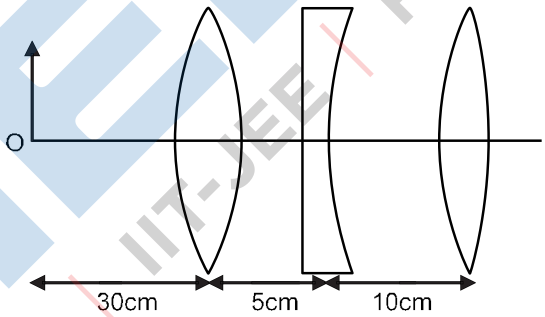
6. If E and H represents the intensity of electric field and magnetising field respectively, then the unit of E/H will be :

(1) newton (2) mho (3) ohm (4) joule

Ans. (3)

Sol. $$\frac{E}{H} = \frac{\frac{F}{q}}{\frac{I}{r}} = \frac{Fr}{I^2 t} = \text{Joule}/\left(\text{second} - \text{Ampere}^2\right)$$

7. Find the distance of the image from object O, formed by the combination of lenses in the figure :



Ans. (2)

Sol. (1) Lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{10},$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{30}; \frac{1}{v} = \frac{30-1}{30}; v = 15cm$$

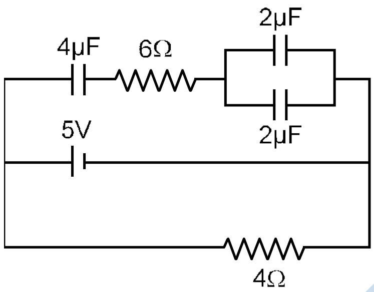
(2) u = +10

$$\frac{1}{v}-\frac{1}{10}-\frac{1}{-10}$$

$$\frac{1}{v}=\frac{-1}{10}+\frac{1}{-10} \quad v=\infty$$

(3) V = +30cm (from third lens)

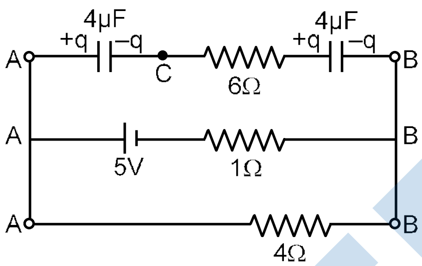
8. Calculate the amount of charge on capacitor of 4 μF. The internal resistance of battery is 1Ω:



(1) Zero (2) 8μC (3) 16μC (4) 4μC

Ans. (2)

Sol.

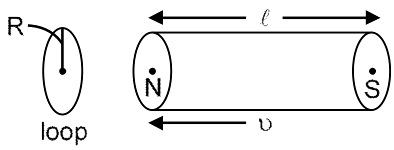


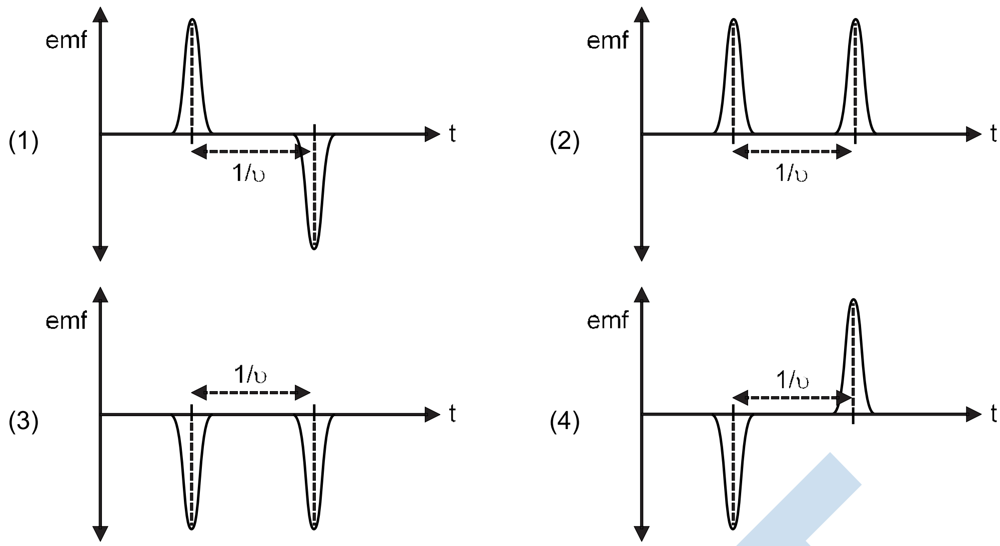
Using $i=\frac{5}{4+1}=1A$

$\therefore V_{AB} = i \times 4 = 4$ V $\therefore V_{AC} = V_{CB} = 2$ Volt

$\therefore$ q on 4μF = CV_{AC} = 8μC

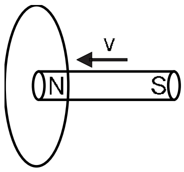
9. A bar magnet is passing through a conducting loop of radius R with velocity v. The radius of the bar magnet is such that it just passes through the loop. The induced e.m.f. in the loop can be represented by the approximate curve :



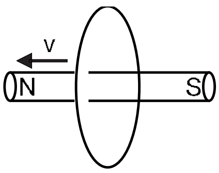


Ans. (4)

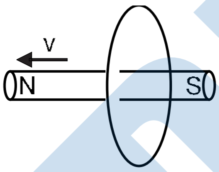
Sol. As Magnet is approaching toward the loop magnetic flux increases and Rate of increment of flux also increases so emf increases with time



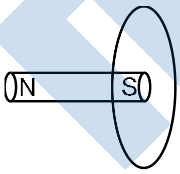
at this moment emf is maximum and negative



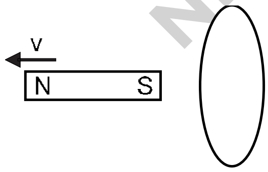
Magnet is in middle of coil, at this moment emf is equal to zero



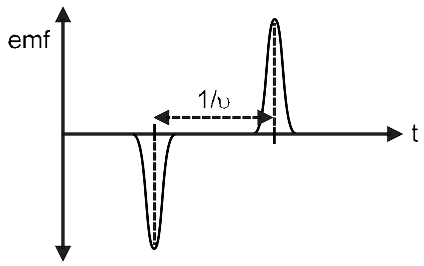
Now again flux start changing however polarity of emf get reverse



At this moment emf is again maximum



emf again become zero

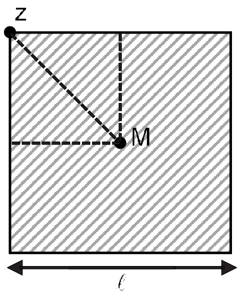


10. Moment of inertia of a square plate of side ℓ about the axis passing through one of the corner and perpendicular to the plane of square plate is given by :

(1) $\frac{2}{3}M\ell^2$ (2) $\frac{M\ell^2}{12}$ (3) $\frac{M\ell^2}{6}$ (4) $M\ell^2$

Ans. (1)

Sol.



$$I_z = I_{cm} + M\left(\frac{\ell}{\sqrt{2}}\right)^2$$

$$= \frac{M\ell^2}{6} + \frac{M\ell^2}{2} = \frac{4M\ell^2}{6} = \frac{2M\ell^2}{3}$$

11. In a photoelectric experiment, increasing the intensity of incident light :

(1) Increases the number of photons incident and the K.E. of the ejected electrons remains unchanged.

(2) Increases the frequency of photons incident and the K.E. of the ejected electrons remains unchanged.

(3) Increases the number of photons incident and also increases the K.E. of the ejected electrons.

(4) Increases the frequency of photons incident and increases the K.E. of the ejected electrons.

Ans. (1)

Sol. $N = \frac{IA}{h\nu}$

12. Two ions of masses 4 amu and 16 amu have charges +2e and +3e respectively. These ions pass through the region of constant perpendicular magnetic field. The kinetic energy of both ions is same. Then :

(1) No ion will be deflected

(2) lighter ion will be deflected less than heavier ion

(3) lighter ion will be deflected more than heavier ion

(4) both ions will be deflected equally.

Ans. (3)

Sol. $r = \frac{P}{qB} = \frac{\sqrt{2mk_e}}{qB}$

$r \propto \frac{\sqrt{m}}{q}$

$\frac{r_1}{r_2} = \frac{3}{4}$

$r_2 > r_1$

13. An ideal gas is expanding such that PT^3 = constant. The coefficient of volume expansion of the gas is :

(1) $\frac{1}{T}$ (2) $\frac{3}{T}$ (3) $\frac{4}{T}$ (4) $\frac{2}{T}$

Ans. (3)

Sol. $\gamma = \frac{1}{V}\frac{dV}{dt}$

PT^3 = constant

$\frac{T}{V}T^3 = \text{Constant}$

$T^4V^{-1} = C$

$V^{-1}(4T^3dT) + T^4\left(\frac{-1}{V^2}dV\right) = 0$

$\frac{dV}{dT} = \frac{4T^3/V}{T^4/V^2} = \frac{4V}{T}$ $\therefore \gamma = \frac{4}{T}$

14. For a transistor in CE mode to be used as an amplifier, it must be operated in :

(1) Cut–off region only (2) The active region only

(3) Saturation region only (4) Both cut–off and Saturation

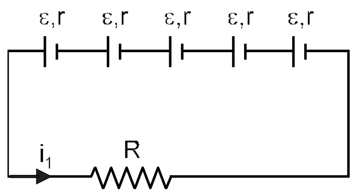
Ans. (2)

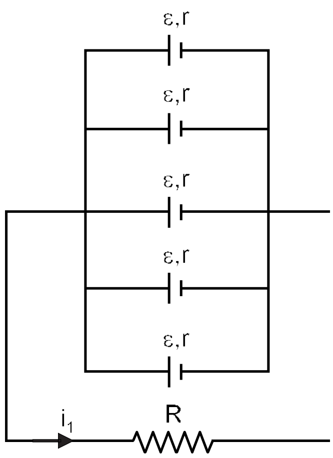
15. Five identical cells each of internal resistance 1Ω and emf 5V are connected in series and in parallel with an external resistance 'R'. For what value of 'R', current in series and parallel combination will remain the same ?

(1) 10 Ω (2) 1 Ω (3) 25 Ω (4) 5 Ω

Ans. (2)

Sol.





$i_1 = \frac{5\varepsilon}{R+5r}$ $\quad$ $i_2 = \frac{\varepsilon}{R+\frac{r}{5}}$

$i_1 = i_2$

$\frac{5\varepsilon}{R+5r} = \frac{\varepsilon}{R+\frac{r}{5}} \Rightarrow R = r = 1\Omega$

16. There are 10^{10} radioactive nuclei in a given radioactive element. Its half–life time is 1 minute. How many nuclei will remain after 30 seconds ? $(\sqrt{2} = 1.414)$

(1) 4×10^{10} (2) 7×10^{9} (3) 10^{5} (4) 2×10^{10}

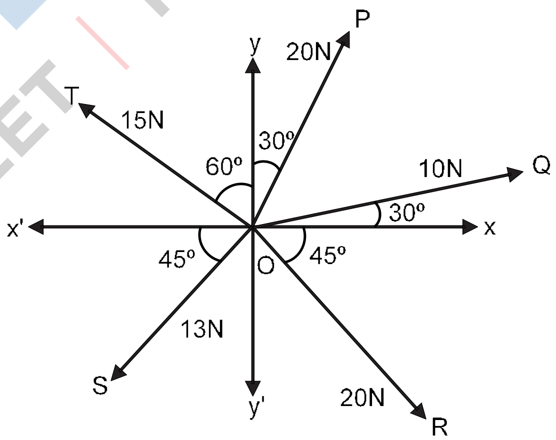
Ans. (2)

Sol. $N = N_0 e^{-\frac{\ell n 2 t}{t_{1/2}}}$

$= 10^{10} e^{-0.35} = 7 \times 10^{9}$

17. The resultant of these forces $\overline{OP}, \overline{OQ}, \overline{OR}, \overline{OS}$ and $\overline{OT}$ is approximately …………….. N.

[Take $\sqrt{3} = 1.7, \sqrt{2} = 1.4$, Given $\hat{i}$ and $\hat{j}$ unit vectors along x, y-axis]



(1) $9.25\hat{i} + 5\hat{j}$ (2) $3\hat{i} + 15\hat{j}$ (3) $-1.5\hat{i} - 15.5\hat{j}$ (4) $2.5\hat{i} - 14.5\hat{j}$

Ans. (1)

Sol. Resultant $(\vec{R}) = \hat{i}$ (10 cos 30° + 20 cos 60° – 15 cos 30° – 15 cos45° + 20 cos 45°)

$+\ \hat{j}$(10 sin 30° + 20 sin 60° + 15 sin 30° – 15 sin 45° – 20 cos 45°)

$= 9.25\hat{i} + 5\hat{j}$

18. Electric field in a plane electromagnetic wave is given by E = 50 sin (500x – 10 × 10^{10} t) V/m. The velocity of electromagnetic wave in this medium is : (Given C = speed of light in vacuum)

(1) $\frac{2}{3}C$ (2) C (3) $\frac{C}{2}$ (4) $\frac{3}{2}C$

Ans. (1)

Sol. $\omega = 10^{10}$

k = 10

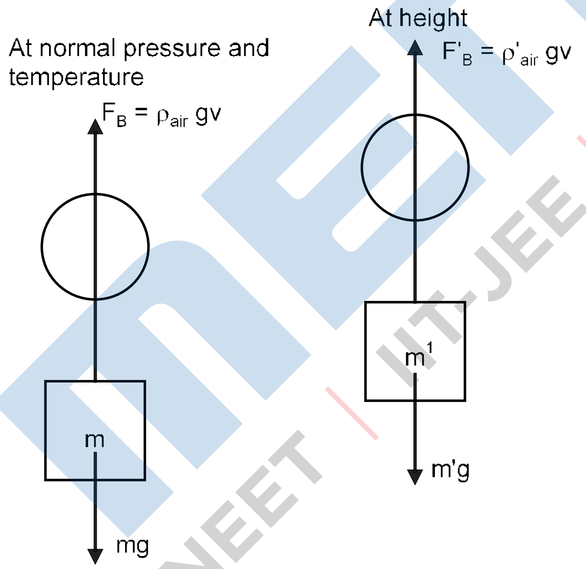
Speed $= \frac{\omega}{K} = \frac{10^{10}}{50} = 2 \times 10^{8} = \frac{2}{3}C$

19. A balloon carries a total load of 185 kg at normal pressure and temperature of 27°C. What load will the balloon carry on rising to a height at which the barometric pressure is 45 cm of Hg and the temperature is –7°C. Assuming the volume constant ?

(1) 214.15 kg (2) 219.07 kg (3) 181.46 kg (4) 123.54 kg

Ans. (4)

Sol.



$\rho_{air}\ gv = mg$

$\rho_{air}\ v = 185$ kg

$\rho_{air}\ gv = m'g$

$\rho_{air}\left(\frac{P'}{P} \times \frac{T}{T'}\right)V = m'$

$185\left[\frac{45}{76} \times \frac{300}{266}\right] = m'$

m' = 123.54 kg

20. An object is placed beyond the centre of curvature C of the given concave mirror. If the distance of the object is d_1 from C and the distance of the image formed is d_2 from C. The radius of curvature of this mirror is :

(1) $\frac{d_1d_2}{d_1+d_2}$ (2) $\frac{2d_1d_2}{d_1+d_2}$ (3) $\frac{d_1d_2}{d_1-d_2}$ (4) $\frac{2d_1d_2}{d_1-d_2}$

Ans. (4)

Sol. $xy = f^2$

$(f + d_1)(f - d_2) = f^2$

$f = \frac{d_1d_2}{d_1-d_2}$

$Roc = 2f = \frac{2d_1d_2}{d_1-d_2}$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. If the velocity of a body related to displacement x is given by $v = \sqrt{5000+24x}$ m/s, then the acceleration of the body is .... m/s^2

Ans. (12)

Sol. $a = \frac{vdv}{dx}$

$v = \sqrt{5000+24x} \times \frac{1}{2\sqrt{5000+24x}} \times 24$

$a = 12\ m/s^2$

2. Two persons A and B perform same amount of work is moving a body through a certain distance d with application of forces acting at angles 45° and 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is ________

Ans. (2)

Sol. $w_1 = w_2$

$F_1\ s \cos 45^\circ = F2\ s \cos 60^\circ$

$\frac{F_1}{F_2} = \frac{1}{\sqrt{2}}$

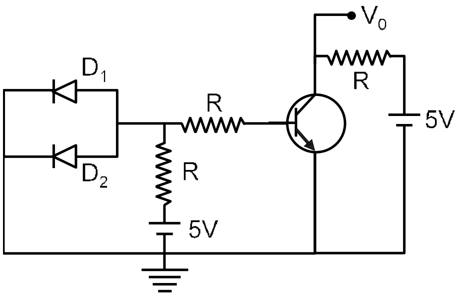
x = 2

3. A transmitting antenna has a height of 320 m and that of receiving antenna is 2000 m. The maximum distance between them for satisfactory communication in line of sight mode is 'd'. The value of 'd' is______ km.

Ans. (224)

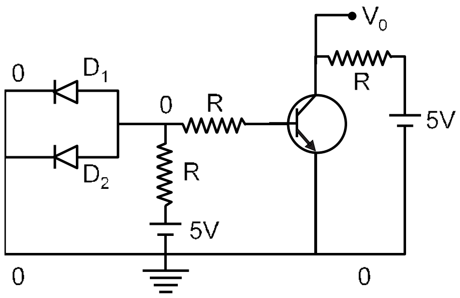
Sol. Range $= \sqrt{2Rh_1} + \sqrt{2Rh_2} = \sqrt{2\times6400\times320} + \sqrt{2\times6400\times200} \approx 224km$

4. A circuit is arranged as shown in figure. The output voltage V0 is equal to ____V.



Ans. (5)

Sol.



Diodes are forward biased, so they act as wire so input current is 0 as input source is short circuited output current is also zero, as input current is zero

Now, output voltage = 5V

5. Two cars X and Y are approaching each other with velocities 36 km/h and 72 km/h respectively. The frequency of a whistle sound as emitted by a passenger in car X, heard by the passenger in car Y is 1320 Hz. If the velocity of sound in air is 340 m/s, the actual frequency of the whistle sound produced is ______Hz.

Ans. (1210)

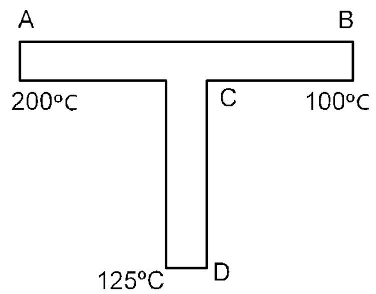
Sol. A ⟶ ⟵ B

V_S = 36 km/hr = 10 m/s V_0 = 72 km/hr = 20 m/s

$$f' = f_0\left(\frac{V+V_0}{V-V_S}\right)$$

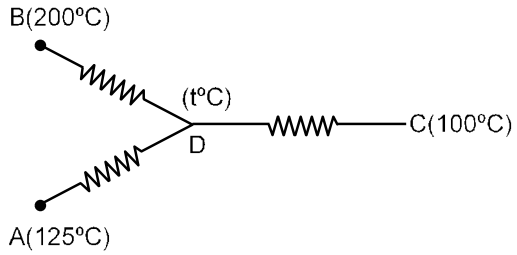
$$f' = 1320\left(\frac{340+20}{340-10}\right) = 1320\times\frac{36}{35} = 1210\text{Hz}$$

6. A rod CD of thermal resistance 10.0 KW^{-1} is joined at the middle of an identical rod AB as shown in figure. The end A, B and D are maintained at 200°C, 100°C and 125°C respectively. The heat current in CD is P watt. The value of P is _______



Ans. (2)

Sol.



$Q_{BD} + Q_{AD} = Q_{DC}$

$$\frac{KA(200-t)}{\frac{\ell}{2}} + \frac{KA(125-t)}{\ell} = \frac{(t-100)KA}{\frac{\ell}{2}}$$

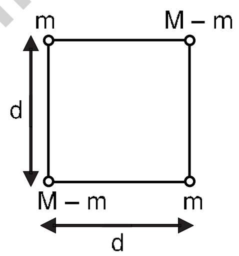
$400 - 2t + 125 - t = 2t - 200$

$725 = 5t$

$t = 145°C$

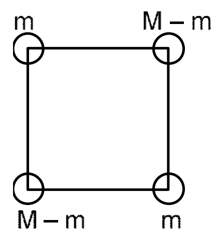
$$Q_{AD} = \frac{KA(125-145)}{\ell} = \left|-\frac{20}{\frac{\ell}{KA}}\right| = -\left|\frac{20}{10}W\right| = 2W$$

7. A body of mass (2M) splits into four masses [m, M–m, m, M–m] which are rearranged to form a square as shown in the figure. The ratio of M/m for which, the gravitational potential energy of the system becomes maximum is x : 1. The value of x is ____________



Ans. (34)

Sol. $U = \frac{Gm^2}{a} + \frac{G(M-m)^2}{a} + \frac{2Gm(M-m)}{a} + \frac{2Gm(M-m)}{\sqrt{2}a}$



For maxima or minima of potential energy $\frac{dU}{dm} = 0$

$\frac{dU}{dm} = \frac{G}{a}(2m - 2(M-m) + (M-2m)2 + \sqrt{2}(M-2m)$

$(4m - 4m - 2\sqrt{2}m) + (-2M + 2M + \sqrt{2}M) = 0\ ;\ \ \sqrt{2}M = 2\sqrt{2}m\ ;\ \frac{M}{m} = 2$

8. First, a set of n equal resistors of 10Ω each are connected in series to a battery of emf 20V and internal resistance 10Ω. A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 20 times, then the value of n is _____.

Ans. (20)

Sol. $i_P = 20 i_s$

$\frac{20}{\left(\frac{10}{n} + 10\right)} = 20\left(\frac{20}{10n+10}\right) \Rightarrow \frac{20n}{10n+10} = 20\left(\frac{20}{10n+10}\right)$

9. The alternating current is given by $i = \left\{\sqrt{42}\sin\left(\frac{2\pi}{T}t\right) + 10\right\}A$. The r.m.s value of this current is ___________ A.

Ans. (11)

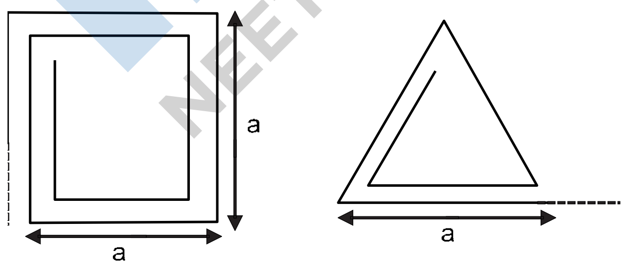
Sol. $i^2 = 42\sin^2\left(\frac{4t}{T}\right) + 100 + 2\sqrt{42}\sin\left(\frac{4t}{T}\right)$

$\langle i^2 \rangle = \frac{42}{2} + 100\ ;\ i_{rms} = \sqrt{\langle i^2 \rangle} = \sqrt{121} = 11$

10. A uniform conducting wire of length is 24a, and resistance R is wound up as a current carrying coil in the shape of an equilateral triangle of side 'a' and then in the form of a square of side 'a'. The coil is connected to a voltage source V_0. The ratio of magnetic moment of the coils in case of equilateral triangle to that for square is $1:\sqrt{y}$ where y is _______

Ans. (3)

Sol.



$4an_1 = 24\,a$ $\qquad$ $3an_2 = 24a \Rightarrow n_2 = 8$ turns thus $n_1 = 6$ turns

$$\frac{M_1}{M_2} = \frac{n_1 i A_1}{n_2 i A_2} = \frac{6 \times a^2}{8 \times a^2 \times \frac{\sqrt{3}}{4}} = \frac{\sqrt{3}}{1}$$

$$\frac{M_2}{M_1} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{y}}$$

$y = 3$

PART B : CHEMISTRY

Single Choice Type

This section contains **20 Single choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Match **List-I** with **List-II**:

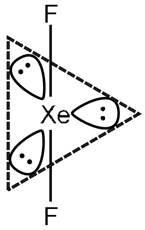
	List-I (Species)		List-II (No. of lone pairs of electrons on the central atom)
(a)	XeF_2	(i)	0
(b)	XeO_2F_2	(ii)	1
(c)	XeO_3F_2	(iii)	2
(d)	XeF_4	(iv)	3

(1) (a) – (iii), (b) – (ii), (c) – (iv), (d) – (i)
(2) (a) – (iii), (b) – (iv), (c) – (ii), (d) – (i)
(3) (a) – (iv), (b) – (i), (c) – (ii), (d) – (iii)
(4) (a) – (iv), (b) – (ii), (c) – (i), (d) – (iii)

Ans. (4)

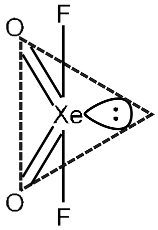
Sol. No. of lp on central atom

(a) XeF_2



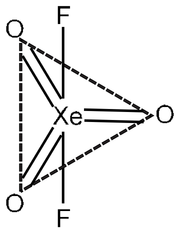
3

(b) XeO_2F_2



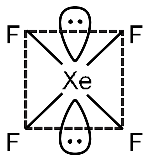
1

(c) XeO_3F_2



0

(d) XeF_4



2

2. The unit of the van der Waals gas equation parameter 'a' in $\left(P+\frac{an^2}{v^2}\right)(V-nb)=nRT$ is:

(1) kg ms^{-2} (2) dm^3 mol^{-1} (3) atm dm^6 mol^{-2} (4) kg ms^{-1}

Ans. (3)

Sol. Pressure correction $\Delta P=\frac{an^2}{V^2}$

$$a=\frac{\Delta P\times V^2}{n^2}\frac{\left(atm\left(dm^3\right)\right)^2}{(mole)^2}=\frac{atm\left(dm^3\right)^2}{(mole)^2}$$

a = atm.dm^6.mol^{-2}

3. Which refining process is generally used in the purification of low melting metals?

(1) Liquation (2) Chromatographic method

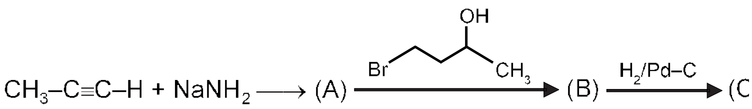
(3) Electrolysis (4) Zone refining

Ans. (1)

Sol. **Liquification of liquation :** In this method a low melting metal like tin can be made to flow on a sloping surface. In this way it is separated from higher melting impurities.

This process is used for the purification of the metal in which melting point of the metal to be purified should be lower than that of each of the impurities associated with the metal. This process is used for the purification of Sn and Zn, and for removing Pb from Zn-Ag alloy.

4. In the following sequence of reactions, the final product D is:

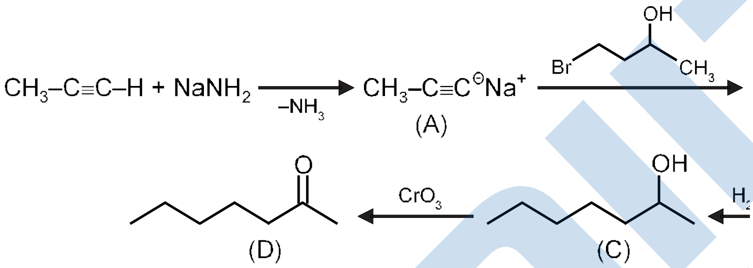


(1) $CH_3–CH_2–CH_2–CH_2–CH_2–\overset{O}{\overset{||}{C}}–CH_3$ (2) $CH_3–CH=CH–CH_2–CH_2–CH_2–COOH$

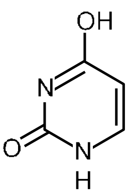
(3) $H_3C–CH_2–CH_2–CH_2–CH_2–\overset{O}{\overset{||}{C}}–H$ (4) $H_3C–CH=CH–CH(OH)–CH_2–CH_2–CH_3$

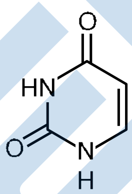
Ans. (1)

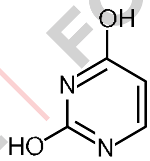
Sol.

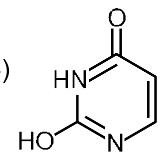


5. Out of following isomeric forms of uracil, which one is present in RNA?

(1) 

(2) 

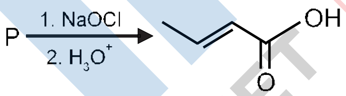
(3) 

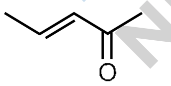
(4) 

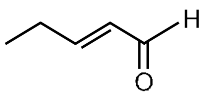
Ans. (2)

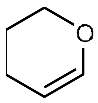
Sol. Fact.

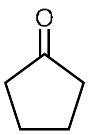
6. The structure of the starting compound **P** used in the reaction given below is:



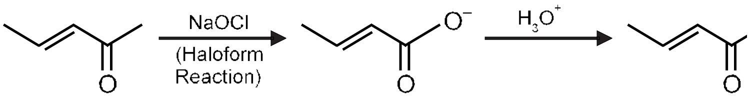
(1) 

(2) 

(3) 

(4) 

Ans. (1)

Sol. 

7. Deuterium resembles hydrogen in properties but:

(1) emits β^+ particles (2) reacts slower than hydrogen

(3) reacts just as hydrogen (4) reacts vigorously than hydrogen

Ans. (2)

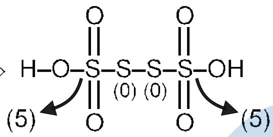
Sol. Reactivity of Deuterium is less than hydrogen due to high bond dissociation energy of D_2

	H_2	D_2
Bond dissociation:	435.88	443.35
Energy (kJ/mole)		

8. In polythionic acid, $H_2S_xO_6$ (x = 3 to 5) the oxidation state(s) of sulphur is/are:

(1) +3 and +5 only (2) 0 and + 5 only (3) +5 only (4) only

Ans (2)

Sol. $H_2S_xO_6 \Rightarrow H_2S_4O_6 \Rightarrow$ 

9. The nature of oxides V_2O_3 and CrO is indexed as "X" and "Y" type respectively. The correct set of X and Y is:

(1) X = basic Y = amphoteric (2) X = amphoteric Y = basic

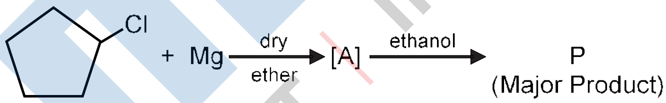
(3) X = basic Y = basic (4) X = acidic Y = acidic

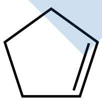
Ans. (3)

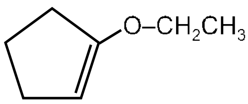
Sol.

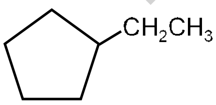
Oxide	Nature
(i) V_2O_3	Basic
(ii) CrO	Basic

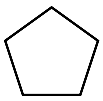
10. In the following sequence of reactions of P is:



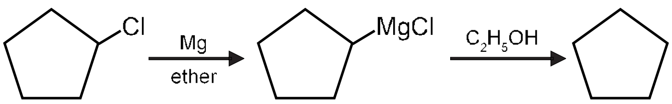
(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. 

11. Match items of **List-I** with those of **List-II**:

List-I (property)		List- II (Example)	
(a)	Diamagnetism	(i)	MnO
(b)	Ferrimagnetism	(ii)	O_2
(c)	Paramagnetism	(iii)	NaCl
(d)	Antiferromagnetism	(iv)	Fe_3O_4

Choose the most appropriate answer from the options given below:

(1) (a) – (i), (b) – (iii), (c) – (iv), (d) – (ii)
(2) (a) – (iv), (b) – (ii), (c) – (i), (d) – (iii)
(3) (a) – (iii), (b) – (iv), (c) – (ii), (d) – (i)
(4) (a) – (ii), (b) – (i), (c) – (iii), (d) – (iv)

Ans. (3)

Sol. MnO - Antiferromagnetism

O_2 - Paramagnetism

NaCl - Diamagnetism

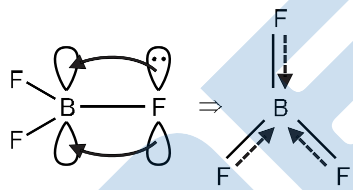
Fe_3O_4 _ Ferrimagnetism

12. In which one of the following molecules strongest back donation of an electron pair from halide to boron is expected?

(1) BI_3 (2) BBr_3 (3) BCl_3 (4) BF_3

Ans. (4)

Sol.



Due to small size atom show strongest back bonding in BF_3

In case of BF_3 (2P – 2P back bonding) while in BCl3 (2P–3P), BBr_3 (2P – 4P) & BI_3 (2P – 5P) back bonding.

So extent of back bonding is maximum in (2P – 2P)

13. The gas 'A' is having very low reactivity reaches to stratosphere. It is non-toxic and non-flammable but dissociated by UV-radiations in stratosphere. The intermediates formed initially from the gas 'A' are:

(1) $CFCl_2$ CH_3 (2) Cl CF_2Cl (3) CH_3 CF_2Cl (4) $Cl\dot{O} + \dot{C}F_2Cl$

Ans. (2)

Sol. $\underset{(CFCs)}{CF_2Cl_2(g)} \xrightarrow{UV} \dot{C}l(g) + \dot{C}F_2Cl(g)$

14. Acidic ferric chloride solution on treatment with excess of potassium ferrocyanide gives a Prussian blue coloured colloidal species. It is:

(1) $K_5Fe[Fe(CN)_6]_2$ (2) $HFe[Fe(CN)_6]$ (3) $Fe_4[Fe(CN)_6]_3$ (4) $KFe[Fe(CN)_6]$

Ans. (4)

Sol. $FeCl_3 + K_4[Fe(CN)_6] \longrightarrow KFe[Fe(CN)_6] + 3KCl$

$FeCl_3$ is limiting reagent. So not all potassium is displaced.

15. Which of the following is **not** a correct statement for primary aliphatic amines?

(1) Primary amines on treating with nitrous acid solution form corresponding alcohols.

(2) Primary amines can be prepared by Gabriel phthalimide synthesis.

(3) The intermolecular association in primary amines is less than the intermolecular association in secondary amines.

(4) Primary amines are less basic than the secondary amines.

Ans. (3)

Sol. The intermolecular association in primary amines is more than the intermolecular association in secondary amines due to 2-H atoms are present on nitrogen.

16. The number of water molecules in gypsum, dead bump plaster and plaster of Paris, respectively are:

(1) 5, 0 and 0.5 (2) 0.5, 0 and 2 (3) 2, 0 and 1 (4) 2, 0 and 0.5

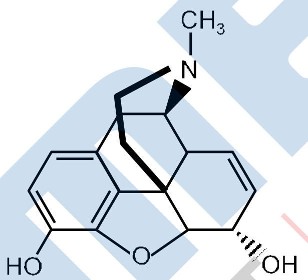
Ans. (4)

Sol. Gypsum = $CaSO_4 \cdot 2H_2O$

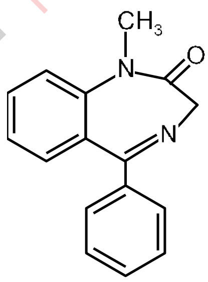
Plaster of paris (POP) = $CaSO_4 \cdot \frac{1}{2}H_2O$

Dead burnt of plaster = $CaSO_4$

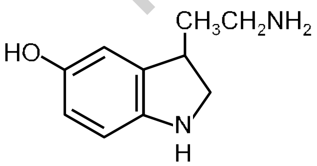
17. (A)



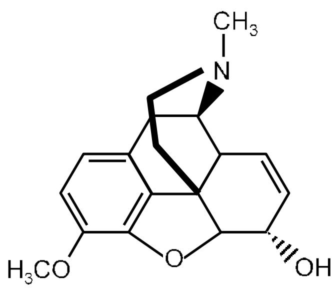
(B)



(C)



(D)



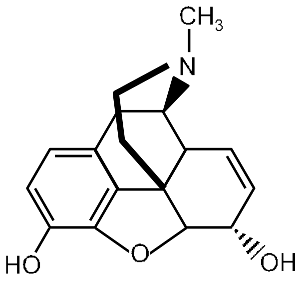
The correct statement about (A), (B), (C) and (D) is:

(1) (B), (C) and (D) are tranquillizers (2) (A), (B) and (C) are narcotic analgesics

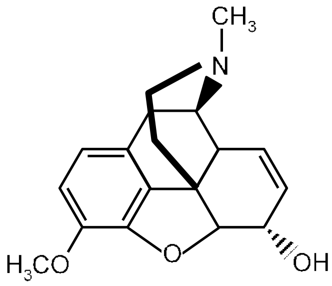
(3) (B) and (C) are tranquilizers (4) (A) and (D) are tranquilizers

Ans. (3)

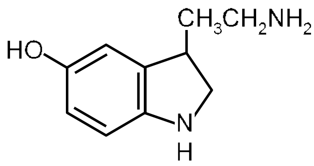
Sol.



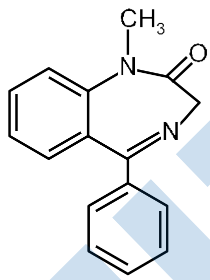
Morphine



Heroin



Seratonin



Valium

Morphine & Heroin are Narcotic analgesic while Valium and Serotonin are tranquilizers.

18. Tyndall effect is more effectively shown by:

(1) true solution (2) suspension (3) lyophilic colloidal (4) lyophobic colloid

Ans. (4)

Sol. Tyndall effect is due to Scattering of light.

Property	Lyophilic colloids	Lyophobic colloids
Tyndall effect	They do not show Tyndall effect	They show Tyndall effect.

19. $CH_3-\underset{\overset{|}{}}{CH}(CH_3)-CH_2-CH_2-\overset{O}{\overset{||}{C}}-Cl \xrightarrow[\text{(iii) } NaNO_2,\ HCl \text{ (iv) } H_2O]{\text{(i) alcoholic } NH_3 \text{ (ii) } NaOH,\ Br_2}$ Major Product

(1) $CH_3-CH(CH_3)-CH_2-CH_2OH$

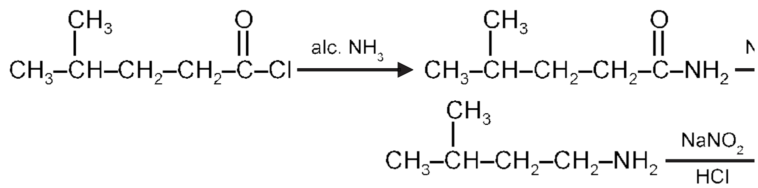
(2) $CH_3-CH(CH_3)-CH_2-CH_2OH$

(3) $CH_3-CH(CH_3)-CH(Br)-CH_2OH$

(4) $CH_3-CH(CH_3)-CH_2-CH_2-Cl$

Ans. (1)

Sol.



20. Given below are two statements: one is labelled as **Assertions (A)** and the other is labelled as **Reason(R).**

Assertions (A) : Synthesis of ethyl phenyl ether may be achieved by Williamson synthesis.

Reason (R) : Reaction of bromobenzene with sodium ethoxide yields ethyl phenyl ether.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **(A)** is correct but **(R)** is not correct.

(2) **(A)** is not correct but **(R)** is correct.

(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**.

(4) Both **(A)** and **(R)** are correct but **(R)** is NOT the correct explanation of **(A)**.

Ans. (1)

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

---

1. In Carius method for estimation of halogens, 0.2 g of an organic compound gave 0.188 g of AgBr. The percentage of bromine in the compound is _________.

[Atomic mass: Ag = 108, Br = 80]

Ans. (40)

Sol. $\frac{\text{Atomic mass of X}}{\text{Molecule mass of Ag}} \times \frac{\text{wt. of Ag X}}{\text{wt of organi chelides}}$

$= \frac{80}{887} \times \frac{0.188}{0.2} \times 100 = 40\%$

2. 200 mL of 0.2 M HCl is mixed with 300 mL of 0.1 M NaOH. The molar heat of neutralization of this reaction is –57.1 kJ. The increase in temperature in °C of the system on mixing is x × 10^{-3}. The value of x is ________.

[Given Specific heat of water = 4.18 J g^{-1} K^{-1} Density of water = 1.00 g cm^{-3}]

(Assume no volume change on mixing)

Ans. (82)

Sol. $HCl + NaCl \longrightarrow NaCl + H_2O$ $\Delta H_{neut} = -57.1$ KJ

	HCl	NaCl	NaCl	H_2O
millimole	40	30	—	—
	0	0	30	—

$\Delta H = [-57.1 \times 30 \times 10^{-3} \times 10^{3}]J = 1713$ J

q = m.s.ΔT

1713 = 500 × 4.18 × ΔT

$\Delta T = 0.8196\ K = 81.96 \times 10^{-2}\ K \approx 82 \times 10^{-2}$ K

3. The reaction that occurs in a breath analyser, a device used to determine the alcohol level in a person's blood stream is

$2K_2Cr_2O_7 + 8H_2SO_4 + 3C_2H_6O \rightarrow 2Cr_2(SO_4)_3 + 3C_2H_4O_2 + 2K_2SO_4 + 11H_2O$

If the rate of appreatance of $Cr_2(SO_4)$ is 2.67 mol min^{-1} at a particular time, the rate of disappearance of C_2H_6O at the same time is ________ mol min^{-1}.

Ans. (4)

Sol. $\frac{1}{3}\frac{-d[C_2H_6O]}{dt} = \frac{1}{2}\frac{d[Cr_2(SO_4)_3]}{dt}$

$\frac{1}{3}$ (Rate of disappearance of C_2H_6O) = $\frac{1}{2}$ (Rate of disappearing of $Cr_2(SO_4)_3$)

Rate of disappearance of C_2H_6O = $\left(\frac{2.67\, mol/min \times 3}{2}\right)$ = 4.005 mol/min

4. 1 mol of an octahedral metal complex with formula $MCl_3.2L$ on reaction with excess of $AgNO_3$ gives 1 mol of AgCl. The denticity of Ligand L is________.

Ans. (2)

Sol. As 1 mole complex give 1 mole AgCl precipitate.

So only one Cl ion is in ionisation sphere.

$\underset{1mole}{ML_2Cl_2\ Cl} \xrightarrow[Excess]{AgNO_3} \underset{1mole}{AgCl}$

As complex is octahedral so denticity of Ligand (L) is = 2.

5. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is equal to $\frac{h^2}{xma_0^2}$. The value of 10 x is ________. (a0 is radius of Bohr's orbit)

[Given : π = 3.14]

Ans. (3155)

Sol. $x = \frac{8\pi^2 \times 16}{4} = 32\pi^2$ $= 32(3.14)^2$ $= 315.5072$

$10x = 315.5072$

6. The number of f electrons in the ground state electronic configuration of Np (Z = 93) is ________.

Ans. (4)

Sol. $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2\,3d^{10}\,4p^6\,5s^2\,4d^{10}\,5p^6\,6s^2\,\boxed{4f^{14}}\,5d^{10}\,6p^6\,7s^4\,\boxed{5f^4}\,6d^1$

$_{93}Np = [_{86}Rn]\,5f^4\,6d^1\,7s^2$

Total no. of electron in f subshell = 14 in 4f and 4 in 5f subshell = $18e^-$

7. The number of moles of CuO, that will be utilized in Dumas method for estimating nitrogen in a sample of 57.5 g of N, N-dimethylaminopentane is __________ $\times 10^{-2}$.

Ans. (1125)

Sol. Moles of N in N,N-dimethylaminopentane = $\frac{57.5}{115}$ 0.5 mol

$$C_7H_{17}N + \frac{45}{2}CuO \longrightarrow 7CO_2 + \frac{17}{2}H_2O + \frac{1}{2}N_2 + \frac{45}{2}Cu$$

$$\frac{\text{no. moles of CuO reacted}}{45/2} = \frac{\text{no. moles of } C_7H_{17}N \text{ reacted}}{1}$$

$\therefore$ no. of moles of CuO reacted $= \frac{45}{2} \times 0.5 = 11.25 = 1125 \times 10^{-2}$

8. When 10 ml of an aqueous solution of $KMnO_2$ was titrated in acidic medium, equal volume of 0.1 M of an aqueous solution of ferrous sulphate was required for complete discharge of colour. The strength of $KMnO_4$ in grams per litre is _________ $\times 10^{-2}$.

[Atomic mass of K = 39, Mn = 53, O = 16]

Ans. (316)

Sol. MnO_4^- + $Fe^{2+} \longrightarrow Mn^{2+} + F^{3+}$

vf = 5 vf = 1

Mili equ. of MnO_4^- = mili equ. of Fe^{2+}

$5[M \times 10] = 1[0.1 \times 10]$

$$M = \left[\frac{0.1}{5}\right] = 0.02\frac{\text{mole}}{\text{Lit}}$$

Strength of $KMnO_4$ = 0.02 × 158 $\frac{\text{gram}}{\text{Lit}}$ = 3.16 g/L = 316 × 10^{-1} g/L

9. The number of moles of NH_3, that must be added to 2L of 0.80 M $AgNO_3$ in order to reduce the concentration of Ag^+ ions to 5.0×10^{-8} M ($K_{formation}$ for $[Ag(NH_3)_2]^+ = 1.0 \times 10^8$) is _______.

[Assume no volume change on adding NH_3]

Ans. (4)

Sol. Let no. moles of NH_3 added = x

$$Ag^+(aq) + 2NH_3(aq) \rightleftharpoons [Ag(NH_3)_2]^+_{(aq)}$$

t = 0	0.8	$\frac{x}{2}$	-
t = ∞	5×10^{-8}	$\left(\frac{x}{2}-1.6\right)$	0.8

$$K_{eq} = \frac{[[Ag(NH_3)_2]^+]}{[Ag^+][NH_3]^2}$$

$$\frac{x}{2}-1.6 = 0.4$$

x = 4

10. 1 kg of 0.75 molal aqueous solution of sucrose can be cooled up to –4ºC before freezing. The amount of ice (in g) that will be separated out is _______. [Given : $K_f(H_2O)$ = 1.86 K kg mol^{-1}]

Ans. (518)

Sol. Let mass of water initially present = x g

Mass of sucrose = (1000 – x) g

Number of moles sucrose $= \frac{1000-x}{342}$

Molality (M) $= \frac{\frac{1000-x}{342}}{x/1000} = 0.75$

2565.5 x = 10^6 – 1000 x

x = 795.86 g

no. of moles of sucrose = 0.5969

$\Delta T_f = ik_f m$

$$4 = 1 \times 1.86 \times \frac{0.5969}{(795.86-a)g} \times 1000$$

ice separated = 518.3 g

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. If 0 < x < 1, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{x}x^4 + ...$, is equal to :

(1) $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

(2) $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$

(3) $\frac{1-x}{1+x} + \log_e(1-x)$

(4) $\frac{1+x}{1-x} + \log_e(1-x)$

Ans. (1)

Sol. Let $t = \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + ...\infty$

$= \left(2-\frac{1}{2}\right)x^2 + \left(2-\frac{1}{3}\right)x^3 + \left(2-\frac{1}{4}\right)x^4 + ...\infty$

$= 2(x^2 + x^3 + x^4 + ...\infty) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...\infty\right)$

$= \frac{2x^2}{1-x} - (\ell n(1-x) - x)$

$\Rightarrow t = \frac{2x^2}{1-x} + x - \ell n(1-x)$

$\Rightarrow t = \frac{x(1+x)}{1-x} - \ell n(1-x)$

2. If for x, y $\in$ **R**, x > 0, $y = \log_{10}x + \log_{10}x^{1/3} + \log_{10}x^{1/9} + ...$ upto ∞ terms and $\frac{2+4+6+....+2y}{3+6+9+....+3y} = \frac{4}{\log_{10}x}$,

then the ordered pair (x, y) is equal to :

(1) $(10^6, 6)$ (2) $(10^4, 6)$ (3) $(10^2, 3)$ (4) $(10^6, 9)$

Ans. (4)

Sol. $\frac{2(1+2+3+....+y)}{3(1+2+3+....+y)} = \frac{4}{\log_{10}x}$

$\Rightarrow \log_{10}x = 6 \Rightarrow x = 10^6$

Now, $y = (\log_{10}x) + \left(\log_{10}x^{\frac{1}{3}}\right) + \left(\log_{10}x^{\frac{1}{9}}\right) + ...\infty$

$= \left(1 + \frac{1}{3} + \frac{1}{9} + ...\infty\right)\log_{10}x$

$= \left(\frac{1}{1-\frac{1}{3}}\right)\log_{10}x = 9$

So, (x, y) = $(10^6, 9)$

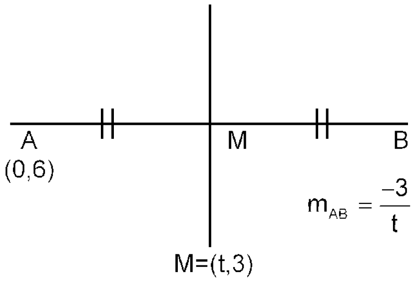
3. Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C.

The locus of the mid-point P of MC is :

(1) $3x^2 - 2y - 6 = 0$ (2) $3x^2 + 2y - 6 = 0$ (3) $2x^2 + 3y - 9 = 0$ (4) $2x^2 - 3y + 9 = 0$

Ans. (3)

Sol. A(0,6) and B(2t,0)



Perpendicular bisector of AB is

$$(y-3)=\frac{t}{3}(x-t)$$

So, $C=\left(0,3-\frac{t^2}{3}\right)$

Let P be (h, k)

$$h=\frac{t}{2};k=\left(3-\frac{t^2}{6}\right)$$

$$\Rightarrow k=3-\frac{4h^2}{6}\Rightarrow 2x^2+3y-9=0$$

4. If $(\sin^{-1}x)^2 - (\cos^{-1}x)^2 = a$; $0 < x < 1$, $a \neq 0$, then the value of $2x^2 - 1$ is :

(1) $\cos\left(\frac{4a}{\pi}\right)$ (2) $\sin\left(\frac{2a}{\pi}\right)$ (3) $\cos\left(\frac{2a}{\pi}\right)$ (4) $\sin\left(\frac{4a}{\pi}\right)$

Ans. (2)

Sol. Given $a = (\sin^{-1} x)^2 - (\cos^{-1} x)^2$

$= (\sin^{-1}x + \cos^{-1}x)(\sin^{-1}x - \cos^{-1}x)$

$$=\frac{\pi}{2}\left(\frac{\pi}{2}-2\cos^{-1}x\right)$$

$$\Rightarrow 2\cos^{-1}x=\frac{\pi}{2}-\frac{2a}{\pi}$$

$$\Rightarrow \cos^{-1}(2x^2-1)=\frac{\pi}{2}-\frac{2a}{\pi}$$

$$\Rightarrow 2x^2-1=\cos\left(\frac{\pi}{2}-\frac{2a}{\pi}\right)$$

5. If the matrix $A=\begin{pmatrix}0 & 2\\ K & -1\end{pmatrix}$ satisfy $A(A^3 + 3I) = 2I$, then the value of K is :

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) -1 (4) 1

Ans. (1)

Sol. Given matrix $A=\begin{bmatrix}0 & 2\\ k & -1\end{bmatrix}$

$A^4 + 3\,IA = 2I$

$\Rightarrow A^4 = 2I - 3A$

Also characteristic equation of A is

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 0-\lambda & 2 \\ k & -1-\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda + \lambda^2 - 2k = 0$

$\Rightarrow A + A^2 - 2K.I$

$\Rightarrow A^2 = 2KI - A$

$\Rightarrow A^4 = 4K^2I + A^2 - 4AK$

Put $A^2 = 2KI - A$

and $A^4 = 2I - 3A$

$2I - 3A = 4K^2I + 2KI - A - 4AK$

$\Rightarrow I(2 - 2K - 4K^2) = A(2 - 4K)$

$\Rightarrow -2I(2K^2 + K - 1) = 2A(1 - 2K)$

$\Rightarrow -2I(2K - 1)(K + 1) = 2A(1 - 2K)$

$\Rightarrow (2K - 1)(2A) - 2I(2K - 1)(K + 1) = 0$

$\Rightarrow (2K - 1)[2A - 2I(K + 1)] = 0$

$$\Rightarrow K = \frac{1}{2}$$

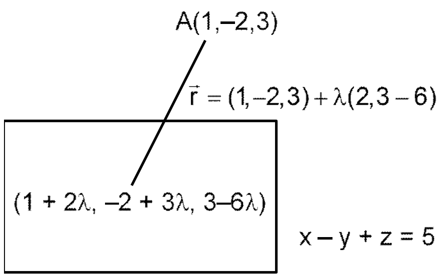
6. The distance of the point (1, –2, 3) from the plane x – y + z = 5 measured parallel to a line, whose direction ratios are 2, 3, –6 is :

(1) 3 (2) 5 (3) 2 (4) 1

Ans. (4)

Sol. $(1 + 2\lambda) + 2 - 3\lambda + 3 - 6\lambda = 5$

$$\Rightarrow 6 - 7\lambda = 5 \Rightarrow \lambda = \frac{1}{7}$$



$$\text{so, } P = \left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$$

$$AP = \sqrt{\left(1-\frac{9}{7}\right)^2 + \left(-2+\frac{11}{7}\right)^2 + \left(3-\frac{15}{7}\right)^2}$$

$$AP = \sqrt{\left(\frac{4}{49}\right) + \frac{9}{49} + \frac{36}{49}} = 1$$

7. If $S = \left\{z \in \mathbb{C} : \frac{z-i}{z+2i} \in \mathbb{R}\right\}$, then :

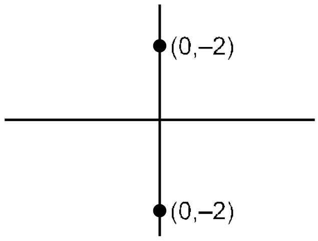
(1) S contains exactly two elements (2) S contains only one element

(3) S is a circle in the complex plane (4) S is a straight line in the complex plane

Ans. (4)

Sol. Given $\frac{z-i}{z+2i} \in R$

Then $\arg\left(\frac{z-i}{z+2i}\right)$ is 0 or Π



$\Rightarrow$ S is straight line in complex

8. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2\sin x - 5)x - 2\cos x$ such that y(0) = 7. Then y(π) is equal to :

(1) $2e^{\pi^2}+5$ (2) $e^{\pi^2}+5$ (3) $3e^{\pi^2}+5$ (4) $7e^{\pi^2}+5$

Ans. (1)

Sol. $\frac{dy}{dx} - 2xy = 2(2\sin x - 5)x - 2\cos x$

$IF = e^{-x^2}$

so, $y.e^{-x^2} = \int e^{-x^2}(2x(2\sin x - 5) - 2\cos x)dx$

$\Rightarrow y.e^{-x^2} = e^{-x^2}(5 - 2\sin x) + c$

$\Rightarrow y = 5 - 2\sin x + c.e^{x^2}$

Given at x = 0, y = 7

$\Rightarrow$ 7 = 5 + c $\Rightarrow$ c = 2

So, $y = 5 - 2\sin x + 2e^{x^2}$

Now at x = π,

y = 5 + 2e $y = 5 + 2e^{x^2}$

9. Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes x – y – z – 1 = 0 and 2x + y – 3z + 4 = 0, is :

(1) 3x – y – 5z + 2 = 0 (2) 3x – 4z + 3 = 0 (3) –x + 2y + 2z – 3 = 0 (4) 4x – y – 5z + 2 = 0

Ans. (4)

Sol. Required equation of plane

$P_1 + \lambda P_2 = 0$

$(x - y - z - 1) + \lambda(2x + y - 3z + 4) = 0$

Given that its dist. From origin is $\frac{2}{\sqrt{21}}$

This $\frac{|4\lambda - 1|}{\sqrt{(2\lambda+1)^2 + (\lambda-1)^2 + (-3\lambda-1)^2}} = \frac{\sqrt{2}}{\sqrt{21}}$

$\Rightarrow 21(4\lambda - 1)^2 = 2(14l2 + 8l + 3)$

$\Rightarrow 336\lambda^2 - 168\lambda + 21 = 28\lambda^2 + 16\lambda + 6$

$\Rightarrow 308\lambda^2 - 184\lambda + 15 = 0$

$\Rightarrow 308\lambda^2 - 154\lambda - 30\lambda + 15 = 0$

$\Rightarrow (2\lambda - 1)(154\lambda - 15) = 0$

$\Rightarrow \lambda = \frac{1}{2} \text{ or } \frac{15}{154}$

for $\lambda = \frac{1}{2}$ reqd. plane is

$4x - y - 5z + 2 = 0$

10. If $U_n = \left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)......\left(1+\frac{n^2}{n^2}\right)^n$, then $\lim_{n\to\infty}(U_n)^{\frac{-4}{n^2}}$ is equal to :

(1) $\frac{e^2}{16}$ (2) $\frac{4}{e}$ (3) $\frac{16}{e^2}$ (4) $\frac{4}{e^2}$

Ans. (1)

Sol. $U_n = \prod_{r=1}^{n}\left(1+\frac{r^2}{n^2}\right)^r$

$\log L = \lim_{n\to\infty}\frac{-4}{n^2}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{n^2}\right)^r$

$\Rightarrow \log L = \lim_{n\to\infty}\sum_{r=1}^{n}-\frac{4r}{n}\cdot\frac{1}{n}\log\left(1+\frac{r^2}{n^2}\right)$

$\Rightarrow \log L \Rightarrow -4\int_0^1 x\log(1+x^2)dx$

put $1 + x^2 = t$

Now, $2xdx = dt$

$= -2\int_1^2 \log(t)dt = -2[t\log t - t]_1^2$

$\Rightarrow \log L = -2(2\log 2 - 1)$

$\therefore\ L = e^{-2(2\log 2 - 1)}$

$= e^{-2\left(\log\left(\frac{4}{e}\right)\right)}$

$= e^{\log\left(\frac{4}{e}\right)^{-2}}$

$= \left(\frac{e}{4}\right)^2 = \frac{e^2}{16}$

11. The statement $(p \wedge (p \to q) \wedge (q \to r)) \to r$ is :

(1) a tautology

(2) equivalent to $p \to \sim r$

(3) a fallacy

(4) equivalent to $q \to \sim r$

Ans. (1)

Sol. $(p \wedge (p \to q) \wedge (q \to r)) \to r$

$\equiv (p \wedge (\sim p \vee q) \vee (\sim q \vee r)) \to r$

$\equiv ((p \wedge q) \wedge (\sim p \vee r)) \to r$

$\equiv (p \wedge q \wedge r) \to r$

$\equiv \sim (p \wedge q \wedge r) \vee r$

$\equiv (\sim p) \vee (\sim q) \vee (\sim r) \vee r$

$\Rightarrow$ tautology

12. Let us consider a curve, $y = f(x)$ passing through the point $(-2, 2)$ and the slope of the tangent to the curve at any point $(x, f(x))$ is given by $f(x) + xf'(x) = x^2$. Then :

(1) $x^2 + 2xf(x) - 12 = 0$

(2) $x^3 + xf(x) + 12 = 0$

(3) $x^3 - 3xf(x) - 4 = 0$

(4) $x^2 + 2xf(x) + 4 = 0$

Ans. (3)

Sol. $y + \frac{xdy}{dx} = x^2$ (given)

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x$

$If = e^{\int \frac{1}{x} dx} = x$

Solution of DE

$\Rightarrow y.x = \int x.x\, dx$

Passes through $(-2, 2)$, ∞

$-12 = -8 + c \Rightarrow c = -4$

$\therefore\ 3xy = x^3 - 4$

i.e. $3x.f(x) = x^3 - 4$

13. $\sum_{k=0}^{20} ({}^{20}C_k)^2$ is equal to :

(1) $^{40}C_{21}$ (2) $^{40}C_{19}$ (3) $^{40}C_{20}$ (4) $^{41}C_{20}$

Ans. (3)

Sol. $\sum_{k=0}^{20} {}^{20}C_k \cdot {}^{20}C_{20-k}$

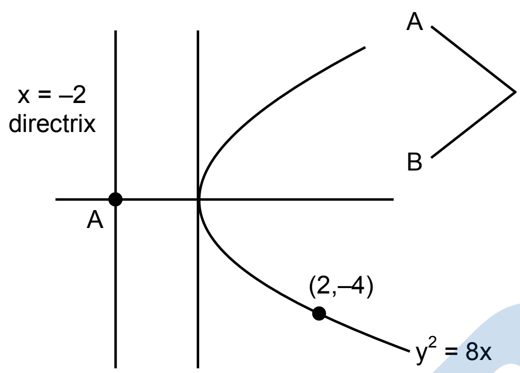
sum of suffix is cont. so summation will be $^{40}C_{20}$

14. A tangent and a normal are drawn at the point p(2, –4) on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the point A and B respectively. If Q(a, b) is a point such that AQBP is a square, then 2a + b is equal to :

(1) – 16 (2) – 18 (3) – 12 (4) – 20

Ans. (1)

Sol.



Equation of tangent at (2, –4) (T = 0)

–4y = 4(x + 2)

x + y + 2 = 0 ....(i)

equation of normal

$x - y + \lambda = 0$

$\downarrow$(2, –4)

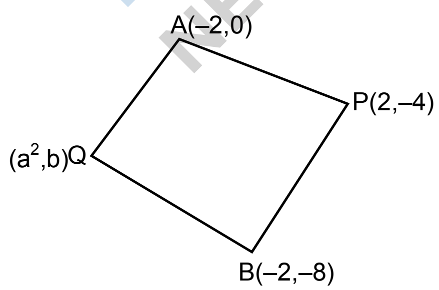
$\lambda = -6$

thus x – y = 6 ….. (2) equation of normal

POI of (1) & x = –2 is A(–2, 0)

POI of (2) & x = –2 is A(–2, 8)

Given AQBP is a sq.



$\Rightarrow \quad m_{AQ} \cdot m_{AP} = -1$

$\Rightarrow \left(\frac{b}{a+2}\right)\left(\frac{4}{-4}\right) = -1 \Rightarrow a+2=b$ ....(1)

Also PQ must be parallel to x-axis thus

$\Rightarrow b = -4$

$\therefore\ a = -6$

Thus $2a + b = -16$

15. Let $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then :

(1) $b^2 - a^2 = a^2 + c^2$ (2) b^2, c^2, a^2 are in A.P.

(3) c^2, a^2, b^2 are in A.P. (4) a^2, b^2, c^2 are in A.P.

Ans. (2)

Sol. $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$

As A, B, C are angles of triangle

$A + B + C = \pi$

$A = \pi - (B + C)$

So, $\sin A = \sin(B + C)$ ....(1)

Similarly $\sin B = \sin(A + C)$ ....(2)

From (1) and (2)

$\frac{\sin(B+C)}{\sin(A+C)} = \frac{\sin(A-C)}{\sin(C-B)}$

$\sin(C + B).\sin(C - B) = \sin(A - C)\sin(A + C)$

$\sin^2 C - \sin^2 B = \sin^2 A - \sin^2 C$

$\{\because \sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y\}$

$2\sin^2 C = \sin^2 A + \sin^2 B$

By sine rule

$2c^2 = a^2 + b^2$

$\Rightarrow b^2, c^2$ and a^2 are in A.P.

16. If α, β are the distinct roots of $x^3 + bx + c = 0$, then $\lim_{x\to\beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2+bx+c)}{(x-\beta)^2}$ is equal to :

(1) $b^2 + 4c$ (2) $2(b^2 + 4c)$ (3) $2(b^2 - 4c)$ (4) $b^2 - 4c$

Ans. (3)

Sol. $\lim_{x\to\beta}\dfrac{1\left(1+\dfrac{2(x^2+bx+c)}{1!}+\dfrac{2^2(x^2+bx+c^2)}{2!}+......\right)-1-2(x^2+bx+c)}{(x-\beta)^2}$

$\Rightarrow \lim_{x\to\beta}\dfrac{2(x^2+bx+1)^2}{(x-\beta)^2}$

$\Rightarrow \lim_{x\to\beta}\dfrac{2(x-\alpha)^2(x-\beta)^2}{(x-\beta)^2}$

$\Rightarrow 2(\beta-\alpha)^2 = 2\ (b^2-4c)$

17. When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6}-x$ and its opposite face occurs with probability $\frac{1}{6}+x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to 7 in any die. If $0<x<\frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is :

(1) $\frac{1}{16}$ (2) $\frac{1}{8}$ (3) $\frac{1}{9}$ (4) $\frac{1}{12}$

Ans. (2)

Sol. Probability of obtaining total sum 7 = probability or getting opposite faces.

Probability of getting opposite faces

$=2\left[\left(\frac{1}{6}-x\right)\left(\frac{1}{6}+x\right)+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\right]$

$=2\left[\left(\frac{1}{6}-x\right)\left(\frac{1}{6}+x\right)+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\right]=\frac{13}{96}$ (given)

$x=\frac{1}{8}$

18. If $x^2 + 9y^2 - 4x + 3 = 0$, x, y $\in \mathbb{R}$, then x and y respectively lie in the intervals :

(1) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and $\left[-\frac{1}{3},\frac{1}{3}\right]$ (2) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and $[1,3]$

(3) [1, 3] and [1, 3] (4) $[1,3]$ and $\left[-\frac{1}{3},\frac{1}{3}\right]$

Ans. (4)

Sol. $x^2 + 9y^2 - 4x + 3 = 0$

$(x^2 - 4x) + (9y^2) + 3 = 0$

$(x^2 - 4x + 4) + (9y^2) + 3 - 4 = 0$

$(x - 2)^2 + (3y)^2 = 1$

$$\frac{(x-2)^2}{(1)^2}+\frac{y^2}{\left(\frac{1}{3}\right)^2}=1$$ (equation of an ellipse).

As it is equation of an ellipse, x & y can vary inside the ellipse.

So, $x-2 \in [-1,1]$ and $y \in \left[-\frac{1}{3},\frac{1}{3}\right]$

$x \in [1,3]$ $y \in \left[-\frac{1}{3},\frac{1}{3}\right]$

19. $\int_6^{16} \frac{\log_e x^2}{\log_e x^2+\log_e(x^2-44x+484)}dx$ is equal to :

(1) 6 (2) 8 (3) 5 (4) 10

Ans. (3)

Sol. Let $I=\int_6^{16} \frac{\log_e x^2}{\log_e x^2+\log_e(x^2-44x+484)}dx$

$I=\int_6^{16} \frac{\log_e x^2}{\log_e x^2+\log_e(x-22)^2}dx$ ....(1)

We know

$\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$ (king)

So $I=\int_6^{16} \frac{\log_e(22-x)^2}{\log_e(22-x)^2+\log_e(22-(22-x))^2}$

$I=\int_6^{16} \frac{\log_e(22-x)^2}{\log_e x^2+\log_e(22-x)^2}dx$ ....(2)

(1) + (2)

$2I=\int_6^{16} 1.dx=10$

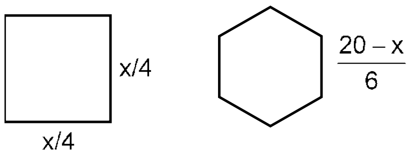
I = 5

20. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made upto a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is :

(1) $\frac{5}{2+\sqrt{3}}$ (2) $\frac{10}{2+3\sqrt{3}}$ (3) $\frac{5}{3+\sqrt{3}}$ (4) $\frac{10}{3+2\sqrt{3}}$

Ans. (4)

Sol. Let the wire is cut into two pieces of length x and 20 – x.



Area of square $=\left(\frac{x}{4}\right)^2$ Area of regular hexagon $=6\times\frac{\sqrt{3}}{4}\left(\frac{20-x}{6}\right)^2$

Total area $=A(x)=\frac{x^2}{16}+\frac{3\sqrt{3}}{2}(20-x)(-1)$

$A'(x)=0$ at $x=\frac{40\sqrt{3}}{3+2\sqrt{3}}$

Length of side of regular Hexagon $=\frac{1}{6}(20-x)$

$=\frac{1}{6}\left(20-\frac{4.\sqrt{3}}{3+2\sqrt{3}}\right)$

$=\frac{10}{2+2\sqrt{3}}$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. Let $\vec{a}=\hat{i}+5\hat{j}+\alpha\hat{k}, \vec{b}=\hat{i}+3\hat{j}+\beta\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{j}-3\hat{k}$ be three vectors such that, $|\vec{b}\times\vec{c}|=5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is _______.

Ans. (90)

Sol. Since, $\vec{a}.\vec{b}=0$

$1 + 15 + \alpha\beta = 0 \Rightarrow \alpha\beta = -16$ ....(1)

Also, $|\vec{b}\times\vec{c}|^2=75 \Rightarrow (10+\beta^2)14-(5-3\beta)^2=75$

$\Rightarrow$ $5\beta^2 + 30\beta + 40 = 0$

$\Rightarrow \quad \beta = -4, -2$

$\Rightarrow \quad \alpha = 4, 8$

$\Rightarrow \quad |\vec{a}|^2_{max} = (26 + \alpha^2)_{max} = 90$

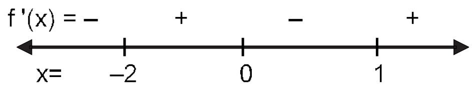
2. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is ________ .

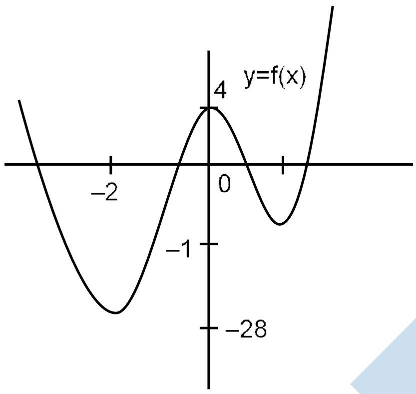
Ans. (4)

Sol. $3x^4 + 4x^3 - 12x^2 + 4 = 0$

$\therefore\ f'(x) = 12x(x^2 + x - 2)$

$= 12x\,(x + 2)\,(x - 1)$





3. Let the equation $x^2 + y^2 + px + (1 - p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2$ and q is an integer$\}$ is _________.

Ans. (61)

Sol. $r = \sqrt{\frac{p^2}{4} + \frac{(1-p)^2}{4} - 5} = \frac{\sqrt{2p^2 - 2p - 19}}{2}$

Since $r \in (0, 5]$

So, $\quad 0 < 2p^2 - 2p - 19 \le 100$

$\Rightarrow p \in \left[\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2}\right) \cup \left(\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2}\right]$

so, number of integer values of p^2 is 61

4. If $A = \{x \in R : |x - 2| > 1\}, B = \left\{x \in R; \sqrt{x^2 - 3} > 1\right\}, C = \{x \in R : |x - 4| \ge 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^C \cap Z$ is ____________.

Ans. (256)

Sol. $A = (-\infty, 1) \cup (3, \infty)$

$B = (-\infty, -2) \cup (2, \infty)$

$C = (-\infty, 2] \cup [3, \infty)$

So, $A \cap B \cap C = (-\infty, -2) \cup [6, \infty)$

$z \cap (A \cap B \cap C)' = \{-2, -1, 0, -1, 2, 3, 4, 5\}$

Hence no. of its subsets = $2^8 = 256$.

5. If $\int \frac{dx}{(x^2+x+1)} = a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C, x > 0$ where C is the constant of integration, then the value of $9\left(\sqrt{3}a+b\right)$ is equal to ________.

Ans. (15)

Sol. $I = \int \frac{dx}{\left[\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right]^2}$

$\int \frac{dt}{\left(t^2+\frac{3}{4}\right)^2}\left(\text{Put } x+\frac{1}{2}=t\right)$

$= \frac{\sqrt{3}}{2}\int \frac{\sec^2\theta d\theta}{\frac{9}{16}\sec^4\theta}\left(\text{Put } t=\frac{\sqrt{3}}{2}\tan\theta\right)$

$= \frac{4\sqrt{3}}{9}\int (1+\cos 2\theta)d\theta$

$= \frac{4\sqrt{3}}{9}\left(\theta+\frac{\sin 2\theta}{2}\right)+c$

$= \frac{4\sqrt{3}}{9}\left[\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{\sqrt{3}(2x+1)}{3+(2x+1)^2}\right]+c$

$= \frac{4\sqrt{3}}{9}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{3}\left(\frac{2x+1}{x^2+x+1}\right)+c$

Hence, $9\left(\sqrt{3}a+b\right)=15$

6. If the system of linear equations

$2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to ________.

Ans. (5)

Sol. 2 × (i) – (ii) – (iii) gives :

$-(1 + \beta)z = 3 - \alpha$

For infinitely many solution

$\beta + 1 = 0 = 3 - \alpha \Rightarrow (\alpha, \beta) = (3, -1)$

Hence, $\alpha + \beta - \alpha\beta = 5$

7. Let n be an odd natural number such that the variance of 1, 2, 3, 4, ….., n is 14. Then n is equal to _________.

Ans. (13)

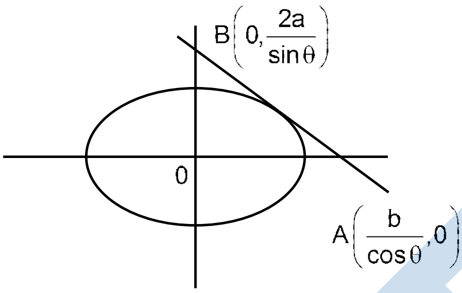
Sol. $\frac{n^2-1}{12} = 14 = 13$

8. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the co-ordinate axis is kab, then k is equal to ___________.

Ans. (2)

Sol. Tangent

$$\frac{x\cos\theta}{b} + \frac{y\sin\theta}{2a} = 1$$



So, $\text{area}(\Delta OAB) = \frac{1}{2} \times \frac{b}{\cos\theta} \times \frac{2a}{\sin\theta}$

$= \frac{2ab}{\sin 2\theta} \geq 2ab$

$\Rightarrow$ k = 2

9. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is ________.

Ans. (100)

Sol.

5	a	b	b	a	5

It is always divisible by 5 and 11.

So, required number = 10 × 10 = 100

10. If $y^{1/4} + y^{-1/4} = 2x$, and $(x^2-1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$, then $|\alpha - \beta|$ is equal to ________.

Ans. (17)

Sol. $y^{\frac{1}{4}} + \frac{1}{y^{\frac{1}{4}}} = 2x$

$$\Rightarrow \left(y^{\frac{1}{4}}\right)^2 - 2xy^{\left(\frac{1}{4}\right)} + 1 = 0$$

$$\Rightarrow y^{\frac{1}{4}} = x + \sqrt{x^2-1} \text{ or } x - \sqrt{x^2-1}$$

So, $$\frac{1}{4}\frac{1}{y^{\frac{3}{4}}}\frac{dy}{dx} = 1 + \frac{x}{\sqrt{x^2-1}}$$

$$\Rightarrow \frac{1}{4}\frac{1}{y^{3/4}}\frac{dy}{dx} = \frac{y^{\frac{1}{4}}}{\sqrt{x^2-1}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{4y}{\sqrt{x^2-1}} \qquad \text{....(1)}$$

Hence, $$\frac{d^2y}{dx^2} = 4\frac{\left(\sqrt{x^2-1}\right)y' - \frac{yx}{\sqrt{x^2-1}}}{x^2-1}$$

$$\Rightarrow (x^2-1)y'' = 4\frac{(x^2-1)y' - xy}{\sqrt{x^2-1}}$$

$$\Rightarrow (x^2-1)y'' = 4\left(\sqrt{x^2-1}\,y' - \frac{xy}{\sqrt{x^2-1}}\right)$$

$$\Rightarrow (x^2-1)y'' = 4\left(4y - \frac{xy'}{4}\right) \quad \text{(from I)}$$

$$\Rightarrow (x^2 - 1)y'' + xy' - 16y = 0$$

So, $|\alpha - \beta| = 17$